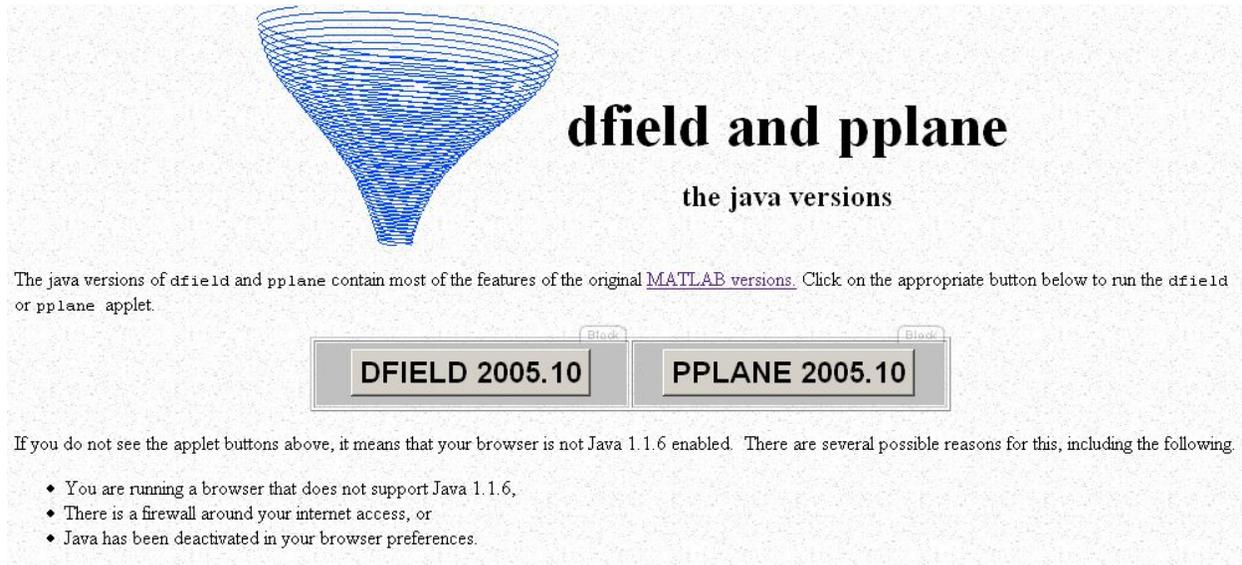


## Using web-based Java pplane applet to graph solutions of systems of differential equations

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Our class project for MA 341 involves using computer tools to analyse solutions of differential equations. This is an introduction to using a java version of the Matlab pplane routine which is far more user friendly than coding in Matlab.

The creator of the Matlab programs pplane and dfield, Dr. John Polking, developed a java version that adds a user interface. This java version can be found at <http://math.rice.edu/~dfield/dfpp.html> or by following the first hit on Google for “pplane”. It’s worth noting that this requires a browser with a Java plug-in, and most of the computer labs on campus do not have this plug-in installed and students don’t have admin rights to install it, so you’ll have to do this on your own computer.



The java versions of `dfield` and `pplane` contain most of the features of the original [MATLAB versions](#). Click on the appropriate button below to run the `dfield` or `pplane` applet.

**DFIELD 2005.10**      **PPLANE 2005.10**

If you do not see the applet buttons above, it means that your browser is not Java 1.1.6 enabled. There are several possible reasons for this, including the following.

- You are running a browser that does not support Java 1.1.6.
- There is a firewall around your internet access, or
- Java has been deactivated in your browser preferences.

The web page of the applet.

If you don’t see the two buttons marked DFIELD and PPLANE, you don’t have Java working.

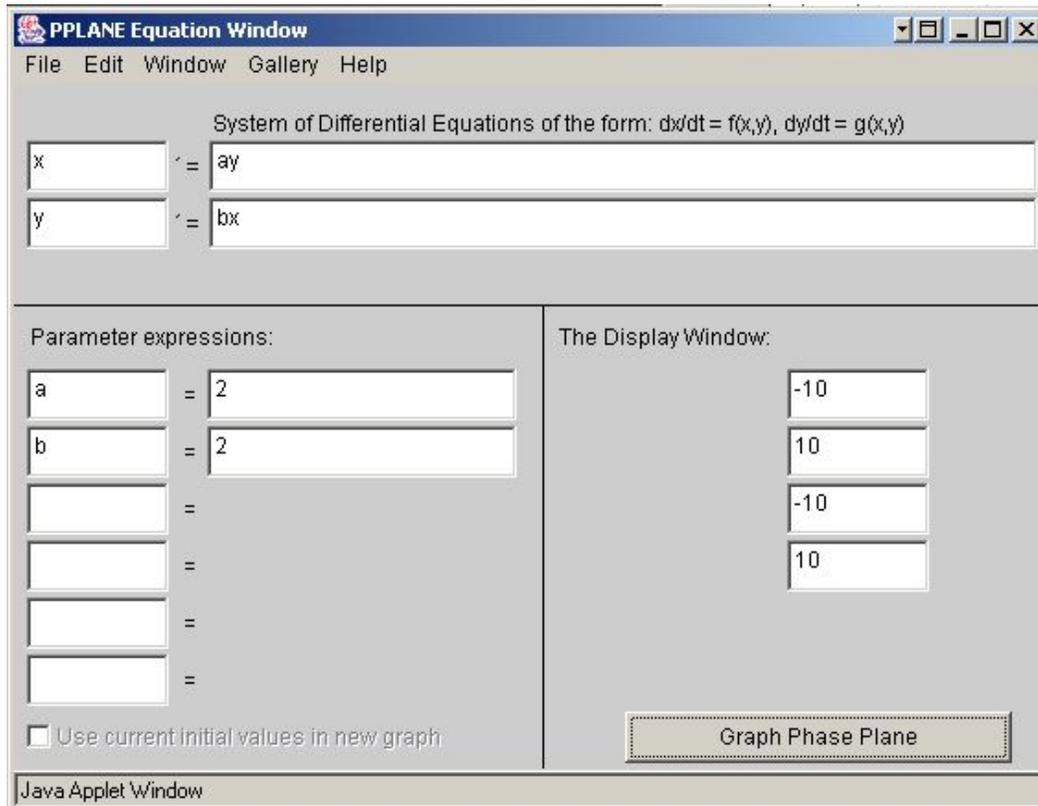
Click on PPLANE. This will bring up four windows: An introduction window, the messages window, the phase plane graph, and the equation window.

Read the introduction window text, and click OK to start using the program.

**The Equation Window** has an example system of first order equations to get you started. At the top you can change the equation. If you want to include parameters, you can define them on the lower right. For example, if you wanted to investigate the effect of different coefficients on the same system of equations, you could enter the equations  $x' = ay$  and  $y' = bx$  at the top, then in the parameter expressions

section, enter values for a, and b. (Note that this is Example 5 on page pp4 of the notes). If a field has invalid syntax it will turn red.

On the lower right of this window, you can set the range of the graph. Once you are happy with your settings, click on “Graph Phase Plane” to update the graph window.

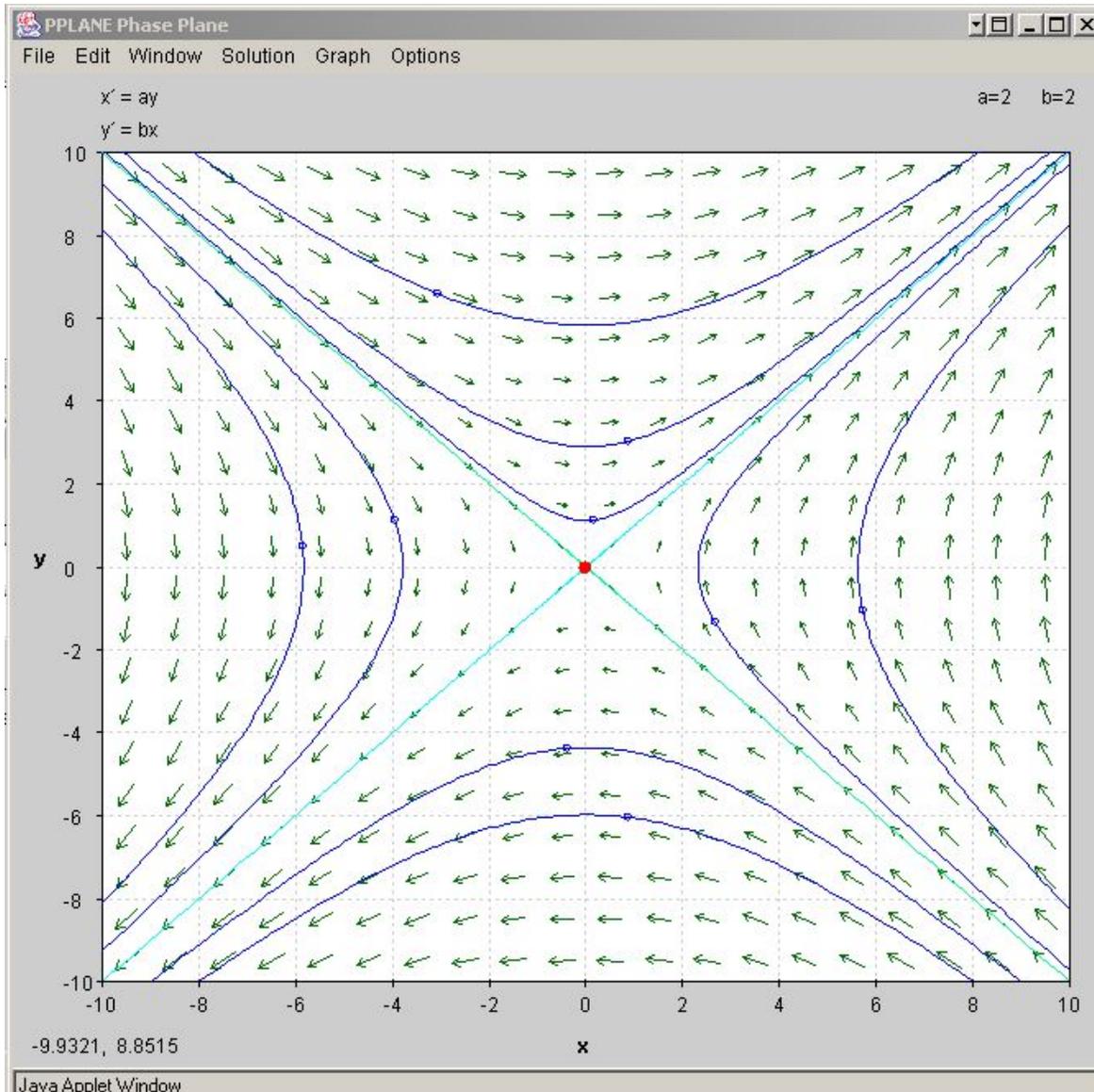


The equation window – eqn entered is E5, pp4.

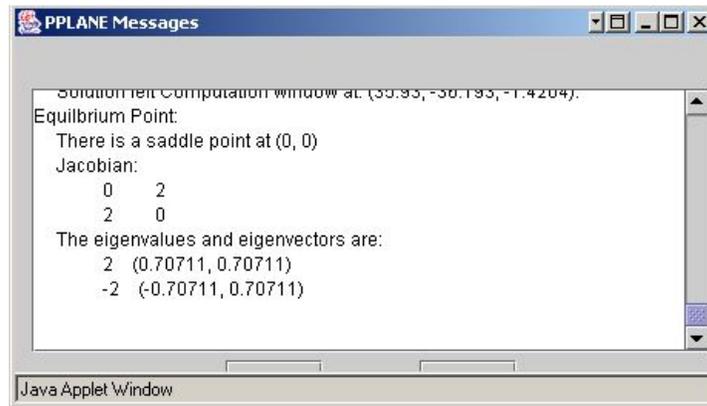
**The Phase Plane Window** displays the graph of the phase plane for the equations and range entered in the Equation window. If you click on the graph, it will plot the trajectory (“orbit”) through that point. The messages window will display the point you selected and where the forward and backward solutions leave the window range.

From the “Solution” menu on this window we can do some interesting things. Choose “Find an equilibrium point” and click on the phase plane graph. The program will solve for the closest equilibrium point and mark it in red. The messages window will display the eigenvalues and eigenvectors and classify the equilibrium point (but not its stability). Note that it chooses eigenvector values such that the magnitude of the vector is 1.

Next choose “plot stable and unstable orbits”, and click on the graph window. The program will solve for trajectories that pass through the nearest equilibrium point, light green for stable, light blue for unstable. (this only works for saddle points I think).



Phase plane plot of E5, pp4



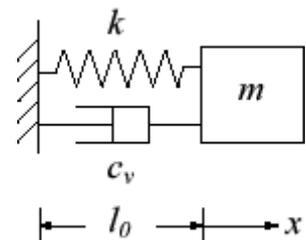
Message window showing result of equilibrium point solution.

Experiment with the options and solutions to get different results in the graph. It's possible to see a 3-d plot of a particular orbit (in  $x, y, t$ ) and also to plot the lines where  $x' = 0$  and  $y' = 0$ .

I found that printing to PDF gave good output. With a pdf printer installed on your computer, choose print in the phase plane window and select the pdf printer. You'll get a pretty nice letter-sized print of the plot.

An application example:

Consider a mass-spring-damper system with a hooke's law spring of stiffness  $k$  and a linear damper with damping constant  $c$ . From hooke's law, the force exerted by the spring is equal to  $-kx$ , and the force exerted by a linear damper is equal to  $-c \frac{dx}{dt}$ , where  $x$  is the displacement from the equilibrium position.



By newton's second law,  $ma_x = \sum F_x$ , which we can write as:

$$mx'' = -kx - cx'$$

Where  $x' = dx/dt$  and  $a = x'' = d^2x/dt^2$

$$\text{In standard form: } x'' + \frac{c}{m}x' + \frac{k}{m}x = 0 \quad (1)$$

We saw in class how an  $n^{\text{th}}$  order ODE can be represented as  $n$   $1^{\text{st}}$  order ODEs. This  $2^{\text{nd}}$  order system can be reformed as two first order equations, which we can use to graph the phase plane of the system's response.

Let  $x' = v$ . It follows that  $x'' = dv/dt$ .

Inserting into (1):

$$\frac{dv}{dt} + \frac{c}{m}v + \frac{k}{m}x = 0$$

Therefore our system of equations in  $v, x$  is:

$$v' = -\frac{c}{m}v - \frac{k}{m}x$$
$$x' = v$$

Where  $c, m, k$  are positive constants. The phase plane graph will have axes of  $x$  and  $y$ , which correspond to the position and velocity of the mass respectively. If we plot a trajectory by clicking on a point, this is equivalent to specifying an initial position and velocity.

PPLANE Equation Window

File Edit Window Gallery Help

System of Differential Equations of the form:  $dx/dt = f(x,y)$ ,  $dy/dt = g(x,y)$

$v$   $' = -(c/m)v - (k/m)x$

$x$   $' = v$

Parameter expressions:

$c = 0$

$k = 5$

$m = 5$

Use current initial values in new graph

The Display Window:

Minimum  $v = -5$

Maximum  $v = 5$

Minimum  $x = -5$

Maximum  $x = 5$

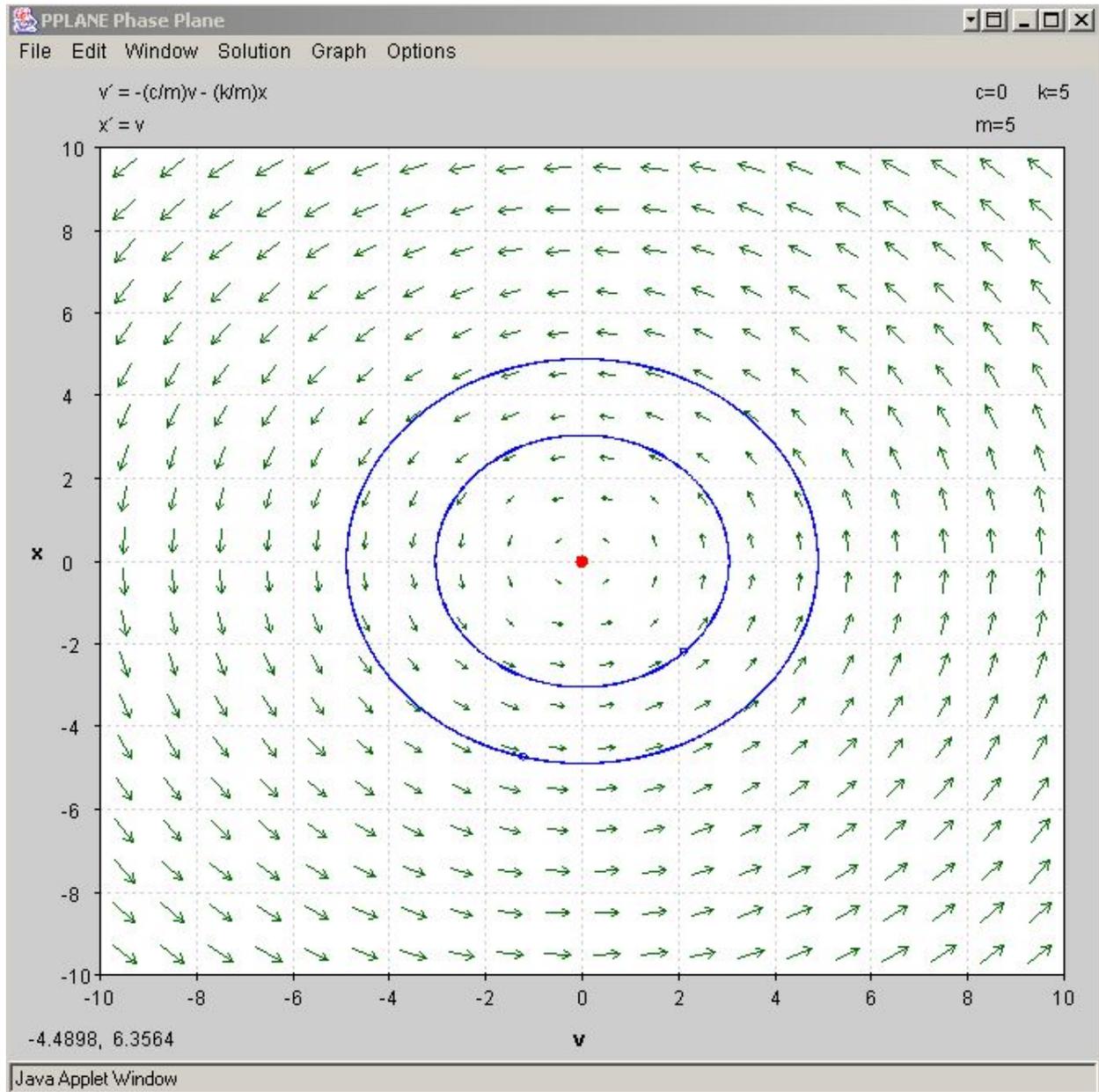
Graph Phase Plane

Java Applet Window

Equation set up for this example.

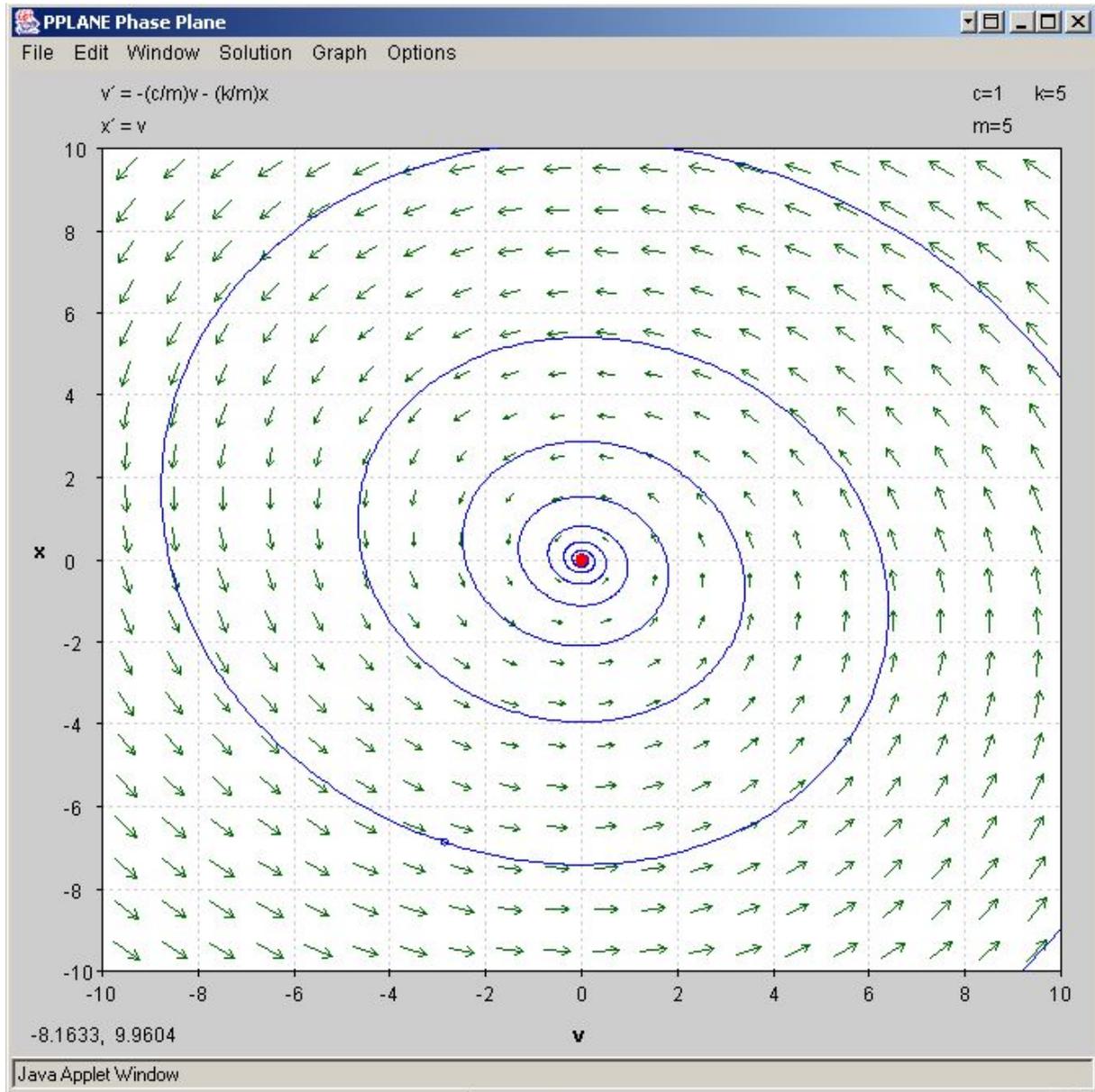
Let's look at two cases of the response here:

- 1)  $C = 0$ : No damping. We'd expect the system to act like an ideal spring, and the solutions should look like closed orbits of the origin. By picking a point in  $(x,v)$  on the graph we are specifying the initial displacement and velocity of the system. For  $c=0, k=5, M = 5$ :



Case 1: No Damping

- 2) With a small damping constant, we expect the system to lose a little velocity every time it goes back and forth. Each cycle should have less and less energy until it stops. For  $c=1$ ,  $k=5$ ,  $m=5$ :



Case 2: Underdamped response

### Relation to the project:

Unfortunately, this relatively painless method only works for the phase plane plotting part of each problem. We'll still need to use something else to plot our calculated solutions and energy analysis plots. That is, #1d, #3c, #4c will need to be plotted in some other software.